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B.E. SEM – 4 (CIVIL)
Question Bank
Maths – II

Each question is of equal Marks (10 Marks)

Q.1	Attempt the following 1) Derive Cauchy –Riemann equations for complex function w = f(z) in polar form. 2) Define harmonic function, Show that $u = \frac{1}{2}\log(x^2 + y^2)$ is harmonic and determine its conjugate function.
Q.2	Attempt the following 1) Derive Cauchy-Rieman equation for a complex function $W = f(z)$ In polar form .Hence deduce that $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$. 2) If $w = u + iv$ represent the complex potential function for an electric field and $u = 3x^2 y - y^3$, determine the function v.
Q.3	Attempt the following 1) If f(z) is analytic function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\lvert f(z)\rvert^2 = 4\lvert f'(z)\rvert^2$ 2) Evaluate $\int_0^{1+i} (x^2 - iy)dz$ along the path $(i) y = x (ii) y = x^2$
Q.4	Attempt the following 1) Find the image of infinite strip $\frac{1}{4} \le y \le \frac{1}{2}$ under the transformation $w = \frac{1}{z}$ also show the region graphically. 2) Define line integral .Evaluate $\int_0^{3+i} z^2 dz$ along (i) the line $y = \frac{x}{3}$; (ii) the parabola $x = 3y^2$
Q.5	Attempt the following 1) Define bilinear transformation, Show that the transformation $w = \frac{2z+3}{z-4}$ maps the circle onto the straight line $4u + 3 = 0$ 2) State Cauchy integral theorem and Cauchy integral formula. Evaluate $\oint_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)}dz$, where $c : \lvert z\rvert = 3$
Q.6	Attempt the following 1) Determine the analytic function whose real part is $u = e^{-x}(x\sin y - y\cos x)$. 2) Find the Bi-linear transformation, which maps the points $z = -1, i, 1$ into the points $w = 1, i, -1$.

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Q.7	Attempt the following 1) Find the Bi-linear transformation which maps the points $z=1,i,-i$ into the Points $w=0,1,\infty$. 2) Use Cauchy's integral formula to evaluate $\int_C \frac{e^{2z}}{(z+1)^4}dz$, where *C* is the circle $\|z\|=2$.
Q.8	Attempt the following 1) Prove that $f(z)=e^{2z}$ is analytic every in the plane and find its derivative. 2) Find the image of $\|z-2i\|=2$ under the mapping $w=\frac{1}{z}$
Q.9	Attempt the following 1) Evaluate $\int_{1-i}^{2+3i}(z^2+z)dz$ along the line joining the points $(1,-1)$ *and* $(2,3)$. 2) Evaluate $\oint_C \frac{2z+1}{z^2+z}dz$; where C *is* $\|z\|=\frac{1}{2}$.
Q.10	Attempt the following 1) If $w=u+iv$ represent the complex potential function for an electric field and $u=3x^2y-y^3$, determine the function v. 2) State the Residue theorem and evaluate $\oint_C \frac{2z+1}{(2z-1)^2}dz$, where *C* is the circle $\|z\|=1$.
Q.11	Attempt the following 1) Prove that $f(z)=e^{2z}$ is analytic every in the plane and find its derivative. 2) Expand $f(z)=\frac{z}{(z+1)(z+2)}$ about $z=-2$.
Q.12	Attempt the following 1) Evaluate $\int_0^{3+i} z^2dz$ along (i) the line $y=\frac{x}{3}$ (ii) the parabola $x=3y^2$ 2) Find the image of the upper half plane under the transformation $w=\frac{z}{i-z}$.
Q.13	Attempt the following 1) Find the Bi-linear transformation which maps the points $z=1,i,-i$ into the Points

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	$w = 0, 1, \infty$. 2) Determine the analytic function whose real part is $y + e^x \cos y$.
Q.14	Attempt the following 1) Evaluate $\int_C \frac{z^2+1}{z(2z+1)} dz$ where C is $\|z\| = 1$. 2) Under the transformation $w = \frac{1}{z}$ find the image of $x^2 - y^2 = 1$.
Q.15	Attempt the following 1) Find the analytic function whose imaginary part is $e^x \sin y$ 2) Under the transformation $w = \frac{1}{z}$ find the image of $\|z - 2i\| = 2$.
Q.16	Attempt the following 1) Evaluate $\int_C \frac{z^2+1}{z(2z+1)} dz$ where C is $\|z\| = 1$. 2) Under the transformation, $w = \frac{1}{z}$ find the image of $x^2 - y^2 = 1$.
Q.17	Evaluate (i) $\oint_c \frac{e^{2z}}{(z+1)^4} dz$; where $c: \|z\| = 4$ (ii) $\oint_c \frac{e^z}{(z+1)^4 (z-2)} dz$; where $c: \|z-1\| = 3$
Q.18	Attempt the following 3) Derive Cauchy –Riemann equations for complex function w = f(z) in polar form. 4) Define harmonic function, Show that $u = \frac{1}{2}\log(x^2 + y^2)$ is harmonic and determine its conjugate function.
Q.19	Attempt the following 3) Derive Cauchy-Rieman equation for a complex function $W = f(z)$ In polar form .Hence deduce that $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$. 4) If $w = u + iv$ represent the complex potential function for an electric field and $u = 3x^2 y - y^3$, determine the function v .
Q.20	Attempt the following

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	3) If f(z) is analytic function of z, prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\|f(z)\|^2 = 4\|f'(z)\|^2$ 4) Evaluate $\int_0^{1+i}(x^2-iy)dz$ along the path $(i) y = x (ii) y = x^2$
Q.21	Attempt the following 3) Find the image of infinite strip $\frac{1}{4}\le y\le\frac{1}{2}$ under the transformation $w=\frac{1}{z}$ also show the region graphically. 4) Define line integral .Evaluate $\int_0^{3+i} z^2 dz$ along (i) the line $y=\frac{x}{3}$; (ii) the parabola $x=3y^2$
Q.22	Attempt the following 3) Define bilinear transformation, Show that the transformation $w=\frac{2z+3}{z-4}$ maps the circle onto the straight line $4u+3=0$ 4) State Cauchy integral theorem and Cauchy integral formula. Evaluate $\oint_c \frac{\sin\pi z^2+\cos\pi z^2}{(z-1)(z-2)}dz$, where $c:\|z\|=3$
Q.23	Attempt the following 3) Determine the analytic function whose real part is $u=e^{-x}(x\sin y - y\cos x)$. 4) Find the Bi-linear transformation, which maps the points $z=-1,i,1$ into the points $w=1,i,-1$.
Q.24	Attempt the following 3) Find the Bi-linear transformation which maps the points $z=1,i,-i$ into the Points $w=0,1,\infty$. 4) Use Cauchy's integral formula to evaluate $\int_C \frac{e^{2z}}{(z+1)^4}dz$, where *C* is the circle $\|z\|=2$.
Q.25	Attempt the following 3) Prove that $f(z)=e^{2z}$ is analytic every in the plane and find its derivative. 4) Find the image of $\|z-2i\|=2$ under the mapping $w=\frac{1}{z}$
Q.26	Attempt the following

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	3) Evaluate $\int_{1-i}^{2+3i}(z^2+z)dz$ along the line joining the points $(1,-1)$ *and* $(2,3)$. 4) Evaluate $\oint_C \frac{2z+1}{z^2+z}dz$; where C *is* $\lvert z\rvert=\frac{1}{2}$.
Q.27	Attempt the following 3) If $w=u+iv$ represent the complex potential function for an electric field and $u=3x^2y-y^3$, determine the function v. 4) State the Residue theorem and evaluate $\oint_C \frac{2z+1}{(2z-1)^2}dz$, where *C* is the circle $\lvert z\rvert=1$.
Q.28	Attempt the following 3) Prove that $f(z)=e^{2z}$ is analytic every in the plane and find its derivative. 4) Expand $f(z)=\frac{z}{(z+1)(z+2)}$ about $z=-2$.
Q.29	Attempt the following 3) Evaluate $\int_0^{3+i} z^2dz$ along (i) the line $y=\frac{x}{3}$ (ii) the parabola $x=3y^2$ 4) Find the image of the upper half plane under the transformation $w=\frac{z}{i-z}$.
Q.30	Attempt the following 3) Find the Bi-linear transformation which maps the points $z=1,i,-i$ into the Points $w=0,1,\infty$. 4) Determine the analytic function whose real part is $y+e^x\cos y$.
Q.31	Attempt the following 3) Evaluate $\int_C \frac{z^2+1}{z(2z+1)}dz$ where C is $\lvert z\rvert=1$. 4) Under the transformation $w=\frac{1}{z}$ find the image of $x^2-y^2=1$.
Q.32	Attempt the following 3) Find the analytic function whose imaginary part is $e^x\sin y$ 4) Under the transformation $w=\frac{1}{z}$ find the image of $\lvert z-2i\rvert=2$.

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Q.33 Attempt the following

3) Evaluate $\int_C \frac{z^2+1}{z(2z+1)}dz$ where C is $|z|=1$.

4) Under the transformation, $w=\frac{1}{z}$ find the image of $x^2-y^2=1$.

Q.34 Evaluate

(i) $\oint_c \frac{e^{2z}}{(z+1)^4}dz$; where $c:|z|=4$

(ii) $\oint_c \frac{e^{z}}{(z+1)^4(z-2)}dz$; where $c:|z-1|=3$

Q.35 Using Newton's interpolation formula, find the values of *f* (22) from the following data.

x	20	25	30	35	40	45
f(x)	354	332	291	260	231	204

Q.36 Using Lagrange's interpolation formula to find *f (5)* from following data.

x	1	2	3	4	7
f(x)	2	4	8	16	128

Q.37 Evaluate $\int_0^6 \frac{dx}{1+x^2}$ with h = 1 by using Simpson's 3/8 rule.

Q.38 Find $y'(0)$ and y'' from the following table.

x	0	1	2	3	4	5
y	4	8	15	7	6	2

Q.39 Solve $y_{n+2}-2y_{n+1}+y_n=3n+4$

Q.40 Solve the following equations by Gauss elimination method.

$$x+4y-z=-5$$
$$x+y-6z=-12$$
$$3x-y-z=4$$

Q.41 Solve the following equations by Gauss-Jordan method.

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	$x+y+z=9$ $2x-3y+4z=13$ $3x+4y+5z=40$
Q.42	Using Newton's forward interpolation formula, find the value of $f(5)$ if
Q.43	Use Lagrange's formula to find the value of y at x = 5 from the following data.
Q.44	Using Picard's method, find the solution of the differential equation $\frac{dy}{dx}=x-y^2$ given that $y(0)=1$ up to second approximation.
Q.45	Using Taylor's series method, find the value of y at x = 0.1 to five places of decimal for the differential equation $\frac{dy}{dx}=x^2y-1; y(0)=1$.
Q.46	using Newton's forward interpolation formula , Find $\sin 52^\circ$ from following data
Q.47	Using Lagrange's interpolation formula find the value of y when x = 10 from the following table

Q.42 data:

x	4	6	8	10	12
f(x)	1	3	8	16	20

Q.43 data:

x	1	2	3	4	7
y	2	4	8	16	128

Q.46 data:

$\sin 45^\circ$	$\sin 50^\circ$	$\sin 55^\circ$	$\sin 60^\circ$
0.7071	0.7660	0.8192	0.8660

Q.47 data:

X	5	6	9	11
y	12	13	14	16

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Q.48	Evaluate $\int_0^6 \frac{dx}{1+x^2}$ with h = 1 by using Simpson's 1/3 rule.
Q.49	Given that X: 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6 Y: 7.989, 8.403, 8.751, 9.129, 9.451, 9.750, 10.031 Find $\frac{dy}{dx} \& \frac{d^2y}{dx^2}$ at y= 1.5
Q.50	Evaluate the line integral $\int_C [(y+3z)dx+(2z+x)dy+(3x+2y)dz]$ where C is the square formed by the lines $y=\pm 1$ *and* $x=\pm 1.$
Q.51	Evaluate the line integral $\int_C [(x^2+xy)dx+(x^2+y^2)dy]$ where C is the square formed by the lines $y=\pm 1$ *and* $x=\pm 1.$
Q.52	Verify Green's theorem for $\oint_C (x+y)dx+2xy\,dy$,Where C is the boundary of the region bounded by $x=0, y=0, x=a, y=b.$
Q.53	Using Green's Theorem, evaluate $\int_C (x^2ydx+x^2dy)$, where C is the boundary described counter clockwise of the triangle with vertices (0, 0) (1, 0) & (1, 1).
Q.54	Verify Green's theorem for $\oint_C (3x-8y^2)dx+(4y-6xy)dy$,Where C is the boundary of the region bounded by x = 0, y = 0 and x + y = 1.
Q.55	Verify Divergence's theorem for $\vec{F}=4xyz\,i-y^2\,j+yz\,k$ over the cube bounded by the planes x = 0, x = 2, y =0, y = 2, z = 0, z = 2.
Q.56	Verify Gauss Divergence theorem for $\vec{F}=(x^2-yz)\hat{i}+(y^2-zx)\hat{j}+(z^2-xy)\hat{k}$ taken over

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	the rectangular parallopied $0 \le x \le a, 0 \le y \le b, 0 \le z \le c.$
Q.57	Evaluate $\iint_S (\nabla \times \overline{F}).d\overline{S}$ using Stoke's theorem when $\overline{F} = (2x - y, -yz^2, -y^2 z)$. Where S is the upper hemisphere of the sphere $x^2 + y^2 + z^2 = a^2$ and C is its boundary.
Q.58	Evaluate $\oint_c \overline{F}.d\overline{r}$ by Stoke's theorem, where $\vec{F} = y^2\hat{i} + x^2\hat{j} - (x+z)\hat{k}$ and c is the boundary of the triangle with vertices at (0,0,0),(1,0,0)&(1,1,0).
Q.59	Evaluate $\oint_C (xydx + xy^2 dy)$ by Stoke's theorem taking C to be a square in the xy-plane with vertices (1,0),(-1,0),(0,1) and (0,-1)
Q.60	Evaluate the line integral $\int_C [(y+3z)dx + (2z+x)dy + (3x+2y)dz]$ where C is the square formed by the lines $y = \pm 1$ *and* $x = \pm 1.$
Q.61	Evaluate the line integral $\int_C [(x^2 + xy)dx + (x^2 + y^2)dy]$ where C is the square formed by the lines $y = \pm 1$ *and* $x = \pm 1.$
Q.62	Verify Green's theorem for $\oint_C (x+y)dx + 2xy\, dy$,Where C is the boundary of the region bounded by $x = 0, y = 0, x = a, y = b.$
Q.63	Using Green's Theorem, evaluate $\int_C (x^2 ydx + x^2 dy)$, where C is the boundary described counter clockwise of the triangle with vertices (0, 0) (1, 0) & (1, 1).
Q.64	Verify Green's theorem for $\oint_C (3x - 8y^2)dx + (4y - 6xy)dy$,Where C is the boundary of the region bounded by x = 0, y = 0 and x + y = 1.
Q.65	Verify Divergence's theorem for $\overrightarrow{F} = 4xyz\, i - y^2\, j + yz\, k$ over the cube bounded by the planes x = 0, x = 2, y =0, y = 2, z = 0, z = 2.

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Q.66	Verify Gauss Divergence theorem for $\vec{F}=\left(x^2-yz\right)\hat{i}+(y^2-zx)\hat{j}+(z^2-xy)\hat{k}$ taken over the rectangular parallopied $0\le x\le a, 0\le y\le b, 0\le z\le c.$
Q.67	Evaluate $\iint_S (\nabla\times\overline{F}).d\overline{S}$ using Stoke's theorem when $\overline{F}=(2x-y,\ -yz^2, -y^2z)$. Where S is the upper hemisphere of the sphere $x^2+y^2+z^2=a^2$ and C is its boundary.
Q.68	Evaluate $\oint_c \overline{F}.d\bar{r}$ by Stoke's theorem, where $\vec{F}=y^2\hat{i}+x^2\hat{j}-(x+z)\hat{k}$ and c is the boundary of the triangle with vertices at (0,0,0),(1,0,0)&(1,1,0).
Q.69	Evaluate $\oint_C (xydx+xy^2dy)$ by Stoke's theorem taking C to be a square in the xy-plane with vertices (1,0),(-1,0),(0,1) and (0,-1)
Q.69	(a) Find the directional derivative of $\phi=2x^2+3y^2+z^2$ at point (2,1,3) in the direction of the vector (1,-2,0). (b) Find the directional derivative of $\phi=xy^2+yz^3$ at point (1,-1,1) in the direction of the vector (1,2,2).
Q.69	(a) Find the directional derivative of $\phi=xy^2+yz^3$ at point (1,-1,1) along the upward normal to the Surface $x^2+y^2+z^2=9$ at (1,2,2). (b) Find the directional derivative of $\frac{1}{r}$ in the direction of $\vec{r}$, where $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$.
Q.70	(a) An electron moves such that its velocity is always perpendicular to its radius vector .Show that its path is circle. (b) Find the velocity and acceleration of the particle which moves along the curve $x=2\sin 3t,\ y=2\cos 3t,\ z=8t,\ \ t>0$. Also find the magnitude of the velocity and acceleration
Q.71	(a) Show that div $(\phi\vec{A})=\phi(div\vec{A})+(grad\phi).\vec{A}$. (b) Prove that div(grad $r^n)=\nabla^2(r^n)=n(n+1)r^{n-2}$, where $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$.

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Q.72	(a) Prove that $\nabla^2\left(\frac{1}{r}\right)=0$, where $\bar{r}=x\bar{i}+y\bar{j}+z\bar{k}$. (b) For a constant vector $\bar{a}$, show that $\nabla\times\left(\frac{\bar{a}\times\bar{r}}{r^3}\right)=-\frac{\bar{a}}{r^3}+\frac{3(\bar{a}\,.\,\bar{r})}{r^5}\bar{r}$,where $\bar{r}=x\bar{i}+y\bar{j}+z\bar{k}$.
Q.73	(a) Prove that $\nabla^2 f(r)=f''(r)+\frac{2}{r}f'(r)$, where $\bar{r}=x\bar{i}+y\bar{j}+z\bar{k}$. (b) Show that the vector field $\vec{F}=\frac{\vec{r}}{r^3}$ is irrotational as well as solenoidal
Q.74	(a) A fluid motion is given by $\vec{v}=(y+z)\hat{i}+(z+x)\hat{j}+(x+y)\hat{k}$. Is the fluid motion irrotational? (b) Show that $\vec{F}=(y^2-z^2+3yz-2x)i+(3xz+2xy)j+(3xy-2xz+2z)k$ is both irrotational and solenoidal
Q.75	Show that $\vec{F}=(y\sin z-\sin x)i+(x\sin z+2yz)j+(xy\cos z+y^2)k$ is irrotational. Find scalar field ϕ such that $\vec{F}=\nabla\phi$
Q.76	Show that $\vec{F}=(6xy+z^3)i+(3x^2-z)j+(3xz^2-y)k$ is irrotational. Find scalar field ϕ such that $\vec{F}=\nabla\phi$